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Correction to "Internal atmospheric gravity waves at ionospheric heights"

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A recent analysis by Zimmerman (1964) exhibits, at high altitudes, a departure of the minimum scale sizes observed in ionospheric winds from the theoretical minimum reported in my title-referenced paper (Hines, 1960, henceforth Paper I). While the departure was in an acceptable direction, in the sense that the theoretical minimum scale sizes were less than the minimum observed scale sizes, a re-examination of the theoretical work seemed to be in order. It became apparent that a computational or plotting error had been made in the preparation of the diagram depicting conditions at 225 km (Fig. 12 of Paper I), and hence in the theoretical curve used by Zimmerman in his comparison with the data. The present note concerns the revisions that are necessitated in Paper I; those necessitated in Zimmerman's comparison have been prepared for submission to the Journal of Geophysical Research.

A corrected version of Fig. 12 of Paper I is presented herewith. The axes denote values of the horizontal wave number (k_x), the real component of the vertical wave number (k_z), and the reciprocal measures of these quantities, namely, the horizontal wavelength ($\lambda_x \equiv 2\pi k_x^{-1}$) and the vertical wavelength ($\lambda_z \equiv 2\pi |k_z^{-1}|$); the negative of the

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vertical wave number is employed, in keeping with the convention of the original text, since the energy of the waves is believed to be propagated upward. The relation between horizontal and vertical wave numbers is frequency dependent, and is illustrated for waves of period 15, 20 and 40 minutes, respectively, by sloping lines labelled in corresponding boxes. A part of the wave spectrum is subject to severe reflection in the middle atmosphere, as indicated, and tends therefore to be missing at ionospheric heights.

The diagram as so far discussed remains unchanged from Fig. 12 of Paper I. A change is required, however, when we come to distinguish those modes of oscillation that are 'quenched' by viscous dissipation from those that are not. The criterion remains as that given by equation (49) of Paper I, or, more accurately, by equation (48), but the corrected computation shows a substantial shift in the position of the demarkation line from the position marked "erroneous boundary" to that marked "corrected boundary". This shift is such as to increase by about an order of magnitude the minimum wavelengths that remain unquenched.

The original error may have been due to a mistake in plotting the curve, or to a computation which employed a kinematic viscosity two orders of magnitude lower than that stated in the text (and adopted again here); it makes little difference now. We must, however, consider the consequences of the revision in its application to related arguments. These arguments are to be found in Section III.3 of Paper I.

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First, it should be noted that no change is required in the general assessment of representative periods and speeds of horizontal propagation, nor in the arguments that indicate a ducting of wave energy below the mesopause. The overall range of periods is virtually unaffected, although the long-period limit, at about 40 minutes, is now caused more by viscous damping than by hydromagnetic damping. (For clarity, the hydromagnetic damping is not illustrated in the accompanying figure; it will be treated at greater length in a paper now in preparation.)

Taken at face value, the increase in the importance now attributed to viscous damping strengthens three earlier arguments. The confinement of the unquenched spectrum to a smaller domain, unshaded in the accompanying figure, adds weight to the view that the rather smooth observed variations in the F region derive from the highly irregular variations in the E region, through a process of upward propagation of wave energy combined with a dissipation of smaller-scale modes. Again, the competing 'cellular wave' hypothesis of Martyn (1950) demands the nearly total reflection of wave energy at heights near or above the F-layer peak, whereas it is now apparent that the wave energy will be dissipated in large measure even if a reflection mechanism can be envisaged. And, finally, the apparent quenching of the waves at heights near the F1 - F2 transition (Heisler, 1958) is more readily accounted for.

On the other hand, and again taking the change at face value, it must be recognized that one point of comparison with the observations suffers slightly. The smallest unquenched horizontal wavelength is increased to 90 km, and with it the smallest horizontal phase speed rises to 6 km/min. The observed speeds range down to about 2 km/min (Munro, 1958). This satisfactorily exceeded the previously estimated theoretical lower limit (0.8 km/min), but it falls below the corrected lower limit.

Neither the favorable nor the unfavorable aspects of the changes are quite so strong as might be inferred from these 'face-value' revisions. The reason for this is the following. The quenching criterion that has been employed here is based on the dissipation of the greater part of the wave energy in the course of one period of oscillation, given no replenishment. It might be restated in more useful physical terms as a criterion for the dissipation of the wave energy in the course of propagation through one wavelength. That is to say, relatively little wave energy should be found in a 'quenched' mode at a distance of one wavelength further into the medium (i.e., obliquely upward) than the level for which the calculation is made. This is a reasonable criterion for marking the level of quenching, although it involves an uncertainty in application that is of the order of a wavelength. This uncertainty is perfectly acceptable when the relevant wavelengths are small, as at E-region heights, but we are now confronted with wavelengths of 100 km or so. The value of the quenching criterion is now more restricted, and about the best we can say is that the transition which is here derived for 225 km should surely have become applicable by (say) 300 km altitude.

This argument may be emphasized by reference to a somewhat different approach to the question of dissipation, presented by Pitteway and Hines (1963). One of the concepts developed there is the occurrence of a level of maximum amplitude for any given mode of oscillation. Such a level arises under the competing influences of an exponential growth of amplitude with height (due to the constancy of energy flux, in the presence of a density decrease) and the ultimately overwhelming depletion of the energy flux due to dissipation. In waves whose vertical wavelength is small compared to the atmospheric scale height, this level of maximum amplitude occurs substantially below the quenching level. In the waves of present concern, however, it does not. Indeed, application of the formulae of Pitteway and Hines reveals that the smallest unquenched vertical wavelength, determined from the accompanying figure, in fact represents a mode of oscillation which is just attaining maximum amplitude in the vicinity of the 225-km level.

In summary, then, Fig. 12 of Paper I requires revision as given here. The related discussion in Section III.3 must be modified as a result, although the requisite modifications cannot be made without great care in application. The validity of the theory as a means of interpreting the observed ionospheric data remains unimpaired.

Acknowledgment

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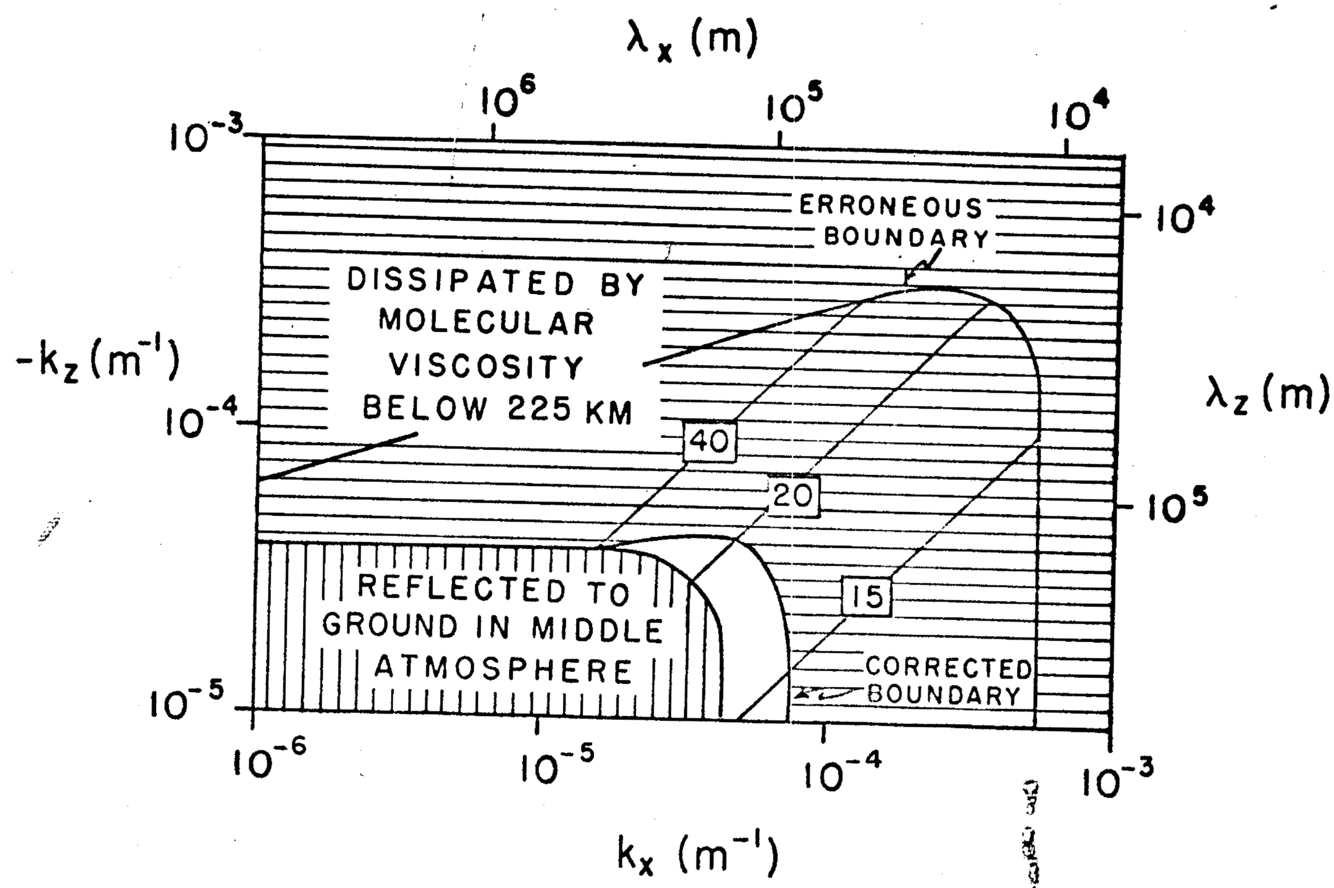
Figure Caption

Correction to Fig. 12 of Paper I.

The horizontal and vertical wave numbers (k_x, k_z) and wavelengths ($\lambda_x \equiv 2\pi k_x^{-1}$, $\lambda_z \equiv 2\pi k_z^{-1}$) provide the horizontal and vertical scales, logarithmically, while internal gravity waves of period 15, 20 and 40 minutes are indicated by truncated oblique lines. Major portions of the k_x - k_z domain are excluded from consideration, as shown by hatching, because of severe reflection in the middle atmosphere or severe damping at higher levels. The unhatched area then represents the modes of greatest importance at heights of 225 km or so, subject to certain approximations discussed in the text and in Paper I.



λ_x (m)
10^6
10^5
10^4
10^{-3}
$-k_z (m^{-1})$
10^{-4}
10^{-5}
λ_z (m)
10^4
10^5
ERRONEOUS BOUNDARY
DISSIPATED BY MOLECULAR VISCOSITY BELOW 225 KM
40
20
15
REFLECTED TO GROUND IN MIDDLE ATMOSPHERE
CORRECTED BOUNDARY
10^{-6}
10^{-5}
10^{-4}
10^{-3}
$k_x (m^{-1})$